

# **AN INTERPOLATION METHOD FOR OBTAINING THERMODYNAMIC PROPERTIES NEAR SATURATED LIQUID AND SATURATED VAPOR LINES**

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## **ABSTRACT**

The two most common approaches used to formulate thermodynamic properties of pure substances are fundamental (or characteristic) equations of state (Helmholtz and Gibbs functions) and a piecemeal approach that is described in Adebisi and Russell (1992). This paper neither presents a different method to formulate thermodynamic properties of pure substances nor validates the aforementioned approaches. Rather its purpose is to present a method to generate property tables from existing property packages and a method to facilitate the accurate interpretation of fluid thermodynamic property data from those tables. There are two parts to this paper. The first part of the paper shows how efficient and usable property tables were generated, with the minimum number of data points, using an aerospace industry standard property package. The second part describes an innovative interpolation technique that has been developed to properly obtain thermodynamic properties near the saturated liquid and saturated vapor lines.

## **INTRODUCTION**

For many years, computer-based mathematical models have been used to analyze thermodynamic systems, and it is no doubt an effective way to save time and money for doing parametric studies and/or what-if scenarios. The quality of a model depends on many factors such as assumptions, governing equations, boundary condition definition, and of course thermodynamic properties. Having good fluid property data not only improves the model's accuracy, but it also makes the model easier to debug. For example, when the modeling results are in question relative to, say, hardware testing results, it is necessary to determine if there are any errors in the model. If the fluid property routines are not trusted, then they may be contributing factors to or the sources of the errors; however, if fluid property routines have been verified, then they are no longer suspects, and the modeler can be confident that any errors are elsewhere.

The fundamental (or characteristic) equations of state and the "piecemeal" method are the two most common approaches used to formulate thermodynamic properties of pure substances. The fundamental equations of state are derived from a combination of the first and second laws of thermodynamics along with the Helmholtz and the Gibbs functions. Using these laws and functions, the complete list of thermodynamic properties can then be obtained by differentiating the fundamental equations of state. On the other hand, the thermodynamic properties equations for the "piecemeal" approach, which are developed by using the specific heat and pressure, specific volume, and temperature data for each phase of a substance, are typically the integrals of the pertinent functions.

Ideally, a simple set of property equations could be integrated into a code (just like conservation of mass, momentum, and energy) and used to obtain every thermodynamic property for all phases of a given substance. Unfortunately, a simple set of property equations does not exist. So the next best approach is to use existing property packages, which contain complex equations with many coefficients, to get the fluid property data necessary for use within

the mathematical models. However, it is very time consuming to use standard property packages within a computer simulation; therefore, property tables are generally used to save computational time. With that in mind, there are two parts to this paper. The first part of the paper shows how efficient and usable property tables were generated, with the minimum number of data points, using an aerospace industry standard property package (based on the fundamental equations of state approach). The second part describes an innovative interpolation technique that has been developed to properly obtain thermodynamic properties near the saturated liquid and saturated vapor lines.

## NOMENCLATURE

$a$  [ft/sec] – speed of sound  
 $C_p$  [Btu/lbm-°R] – constant pressure specific heat  
 $C_v$  [Btu/lbm-°R] – constant volume specific heat  
 $H$  [Btu/lbm] – enthalpy, input enthalpy  
 $H_2$  [Btu/lbm] – second enthalpy value  
 $H_{crit}$  [Btu/lbm] – critical enthalpy  
 $H_{hi}$  [Btu/lbm] – higher enthalpy value of the interpolation rectangle  
 $H_L$  [Btu/lbm] – saturated liquid enthalpy  
 $H_{lo}$  [Btu/lbm] – lower enthalpy value of the interpolation rectangle  
 $H_{min}$  [Btu/lbm] – lowest enthalpy value  
 $H_{Phi}$  [Btu/lbm] – saturated enthalpy for the higher pressure value of the interpolation rectangle  
 $H_{Plo}$  [Btu/lbm] – saturated enthalpy for the lower pressure value of the interpolation rectangle  
 $H_V$  [Btu/lbm] – saturated vapor enthalpy  
 $k$  [Btu/hr-ft-°R] – thermal conductivity  
 $M$  – number of times  $\Delta P$  has been incremented  
 $m_L$  [lbm] – liquid mass  
 $m_V$  [lbm] – vapor mass  
 $N$  – number of times  $\Delta P$  has been incremented  
 $P$  [psi] – pressure, input pressure  
 $P_{avg}$  [psi] – average pressure  
 $P_{Hhi}$  [psi] – pressure at which the saturated liquid line or saturated vapor line crosses the higher enthalpy value of the interpolation rectangle  
 $P_{hi_{crit}}$  [psi] – higher pressure value of the pressure band around the critical point  
 $P_{lo_{crit}}$  [psi] – lower pressure value of the pressure band around the critical point  
 $T$  [°R] – temperature  
 $V$  [ft<sup>3</sup>/lbm] – specific volume  
 $x$  – input into the linear interpolation equation  
 $X$  – quality  
 $x_1$  – first independent variable of the linear interpolation equation  
 $x_2$  – second independent variable of the linear interpolation equation  
 $y$  – output from the linear interpolation equation  
 $y_1$  – first dependent variable of the linear interpolation equation  
 $y_2$  – second dependent variable of the linear interpolation equation  
 $Z$  – a generic property  
 $\Delta H$  [Btu/lbm] – small enthalpy increment  
 $\Delta P$  [psi] – small pressure increment  
 $\gamma$  – specific heat ratio  
 $\rho$  [lbm/ft<sup>3</sup>] – density  
 $\rho_{Hhi}$  [lbm/ft<sup>3</sup>] – density at the higher enthalpy value

$\rho_{H10}$  [lbm/ft<sup>3</sup>] – density at the lower enthalpy value

## RESULTS AND DISCUSSION

### PROPERTY PACKAGES

There are a number of property packages commercially available. NIST, MIPROPS, and GASPAK are a few examples of those packages, and they were used in this paper for comparison testing. As a demonstration of the techniques discussed in this paper, selected sets of data points for oxygen were chosen to represent every part of an entire fluid region. Non-saturated properties were generated using pressure (P) and temperature (T) as inputs, while saturated properties were obtained using pressure and quality (X). Table 1 compares density ( $\rho$ ) and constant pressure specific heat ( $C_p$ ) from the three property packages listed above, and as expected, the thermodynamic properties for those data points are very similar. Thus, for this example, which property package to use to generate property tables is a matter of personal preference. GASPAK was used here due to its relative ease of implementation.

Table 1. Property Data for Different Property Packages

		NIST	MIPROPS	GASPAK
P = 220 psia; X = 0 Saturated Liquid	$\rho$ (lbm/ft <sup>3</sup> )	57.6702	57.6710	57.6687
	$C_p$ (Btu/lbm-°R)	0.4951	0.4950	0.4951
P = 220 psia; X = 1 Saturated Vapor	$\rho$ (lbm/ft <sup>3</sup> )	3.6691	3.6760	3.6688
	$C_p$ (Btu/lbm-°R)	0.3548	0.3500	0.3548
P = 520 psia; T = 200 °R Subcooled Liquid	$\rho$ (lbm/ft <sup>3</sup> )	64.9515	65.0110	64.9514
	$C_p$ (Btu/lbm-°R)	0.4238	0.4220	0.4238
P = 70 psia; T = 500 °R Superheated Vapor	$\rho$ (lbm/ft <sup>3</sup> )	0.4192	0.4200	0.4192
	$C_p$ (Btu/lbm-°R)	0.2208	0.2210	0.2208
P = 5000 psia; T = 700 °R Supercritical	$\rho$ (lbm/ft <sup>3</sup> )	19.6892	19.7160	19.6889
	$C_p$ (Btu/lbm-°R)	0.2755	0.2760	0.2755

### PROPERTY TABLES

The first step in creating computationally efficient fluid property tables involves the evaluation of the property state space. Real fluid property tables were generated using the GASPAK property package (based on fundamental equations of state approach) by inputting pressures and enthalpies. The fundamental goal of this effort was to minimize the number of pressure-enthalpy pairs necessary to generate property tables that fully described the fluid and thus increase computational (interpolation) efficiency. To accomplish this, a computer program was created which indirectly allows the user to determine how many pressure-enthalpy pairs are necessary in order to achieve a prescribed interpolation error. Thus, the desired accuracy of the results across the region of fluid properties considered is used to determine the expanse of the tables generated.

First, the program evaluates fluid properties along the saturated liquid and vapor lines to determine the necessary number of pressure points and their values so that the interpolation error along these lines will be within the specified error. This is illustrated in Figure 1 (a) where the constant P dashed lines cross the saturated liquid and vapor lines at points which allow interpolation for saturated properties within the prescribed error. Then the fluid property data is

divided into two regions, sub-critical and supercritical, for further evaluation. After obtaining the pressure values for saturation lines, the program then determines pressure-enthalpy values for the supercritical region. This is a more involved process than what have been done for the saturation lines, but more or less, the methodology remains functionally similar. The program starts out with the lowest enthalpy value and determines the necessary number of pressure points for the supercritical region so that the interpolation error along that enthalpy line will be within the specified error. Similarly, the program starts out with the lowest pressure point for the supercritical region and determines the necessary number of enthalpy points required to fulfill the specified interpolation error limits. The process is repeated up and across the entire supercritical region until the enthalpy-pressure grid is generated as shown in Figure 1 (b). This same technique could be used for the sub-critical region, and one would find that the sub-critical enthalpy points are almost identical to the supercritical enthalpy points. So for the sub-critical region, it is acceptable to use the same pressure values as the saturation lines, and likewise, the same enthalpy values as the supercritical region. The entire grid is shown in Figure 1 (c).

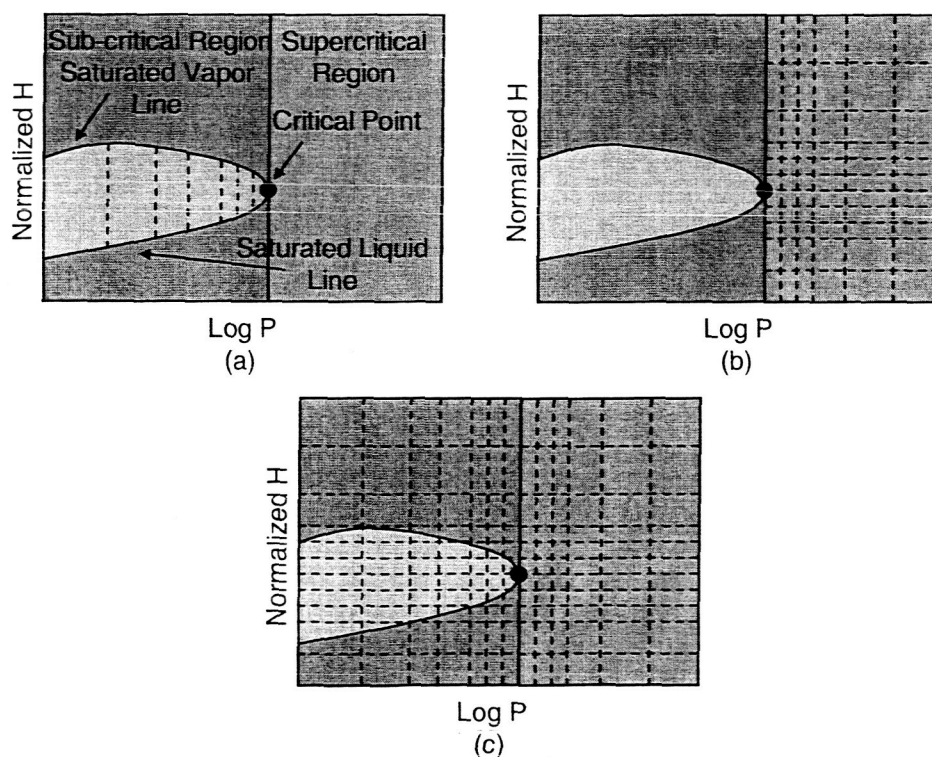


Figure 1. Pressure-Enthalpy Grid

### Linear Interpolation

Throughout this document when the word "interpolation" or its variations are mentioned, they refer to a linear interpolation method that is described by Equation (1),

$$y = (y_2 - y_1) \left( \frac{x - x_1}{x_2 - x_1} \right) + y_1 \quad \text{Equation (1)}$$

where  $x_1$  and  $y_1$  are the first independent value and its corresponding dependent value, respectively.  $x_2$  and  $y_2$  are the second independent and dependent values.  $x$  is the input value, and  $y$  is the output value. As mentioned earlier, property tables are functions of pressure (P)

and enthalpy (H), so for a density table, pressure and enthalpy are both independent variables, and density ( $\rho$ ) is the dependent variable. Now for example, when it states that "the density is determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties using the input enthalpy", it means that the density at a particular pressure P is calculated by using Equation (1) with the following values

$$\rho = (\rho_{H_{hi}} - \rho_{H_{lo}}) \left( \frac{H - H_{lo}}{H_{hi} - H_{lo}} \right) + \rho_{H_{lo}} \quad \text{Equation (2)}$$

where  $\rho_{H_{hi}}$  is the density at the higher enthalpy,  $\rho_{H_{lo}}$  is the density at the lower enthalpy,  $H_{hi}$  is the higher enthalpy,  $H_{lo}$  is the lower enthalpy, H is the input enthalpy, and  $\rho$  is the output density.

### Pressure Values for Saturated Liquid and Vapor Lines

Before using the program to generate property tables, a user must provide a few required inputs as listed below:

1. Ranges of pressures and enthalpies (hence temperatures) that the tables cover.
2. An acceptable error between interpolated table values and GASPAC values.
3. The thermodynamic properties which are checked to make sure that all of the errors for those properties are within the specified value.
4. The critical pressure.
5. The pressure band and enthalpy band around the critical point where the code can not define errors using GASPAC properties because GASPAC would either crash or return unreasonable values.

Next the program evaluates the specified thermodynamic properties along the saturated liquid and saturated vapor lines to determine the necessary number of pressure points and their values so that the interpolation errors along these lines will be within the specified error. As shown in Figure 2, the program calls GASPAC to get saturated liquid (shaded circle) and saturated vapor (unshaded circle) properties for the lowest pressure (P). Then the pressure is incremented by a small delta pressure ( $\Delta P$ ), and saturated properties (shaded and unshaded squares) are then obtained with GASPAC. Now, new properties (shaded and unshaded triangles) are determined by interpolating between the properties at P and at  $P + \Delta P$  using the average pressure of P and  $P + \Delta P$  ( $P_{avg}$ ) as input. By using the same average pressure, the code uses GASPAC to get properties at  $P_{avg}$  for both saturated liquid and saturated vapor. Subsequently the errors can be determined between GASPAC and interpolated properties. If all of the errors for the selected thermodynamic properties are less, by an allowable tolerance, than the specified error then the pressure is again incremented by  $\Delta P$ . So now the new pressure will be  $P + 2\Delta P$ , and interpolation is between P and  $P + 2\Delta P$ . The process repeats until at least one of the errors for the selected properties is greater than the specified error.

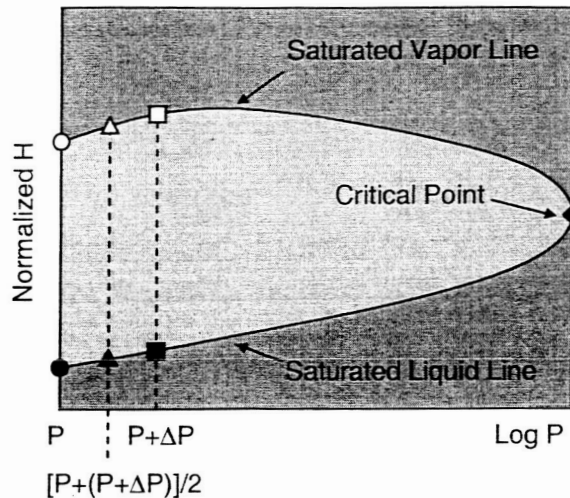


Figure 2. Second Pressure Point for Saturation Lines – Incremented by  $1-\Delta P$

$N$  is the number of times that  $\Delta P$  has been incremented when one of the errors is greater than the specified error. As shown in Figure 3, the current pressure is  $P+N(\Delta P)$ , and the previous pressure is  $P+(N-1)(\Delta P)$ . Those are the upper bound and lower bound, respectively, of the pressure of interest. So the next code step is to go back to the lower bound pressure and increment it by one-tenth of the  $\Delta P$ , as the new  $\Delta P$ , and repeat error determination and keep incrementing the pressure until, again, at least one of the errors for the selected properties is greater than the specified error. When this condition is met, the code goes back to the previous pressure ( $P+(N-1)(\Delta P)+(M-1)(\Delta P/10)$ ), where  $M$  is the number of times  $\Delta P/10$  (the current  $\Delta P$ ) has been incremented, and repeats this iteration process with increments one-tenth of the current  $\Delta P$  (or one-hundredth of the original  $\Delta P$ ). The same process repeats until the maximum error of the selected properties is within the allowable tolerance on the specified error or until the current  $\Delta P$  is less than  $1 \times 10^{-4}$ . If the maximum error is within the allowable tolerance of the specified error then the second pressure point for the saturation lines is the current pressure of the current iteration process. On the other hand, if the current  $\Delta P$  is less than  $1 \times 10^{-4}$  then the second pressure point for the saturation lines is the pressure prior to exceeding the error with this current  $\Delta P$ .

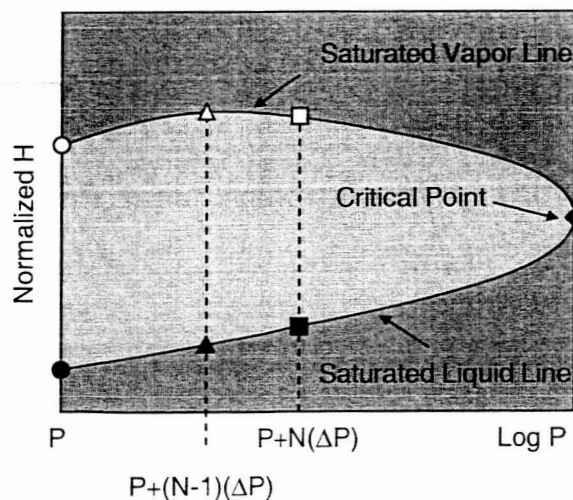


Figure 3. Second Pressure Point for Saturation Lines – Incremented by  $N-\Delta P$

Starting from the just determined pressure point, the iteration process repeats to determine the other pressure points for the saturation lines. However, the code only goes up to the lower end of the pressure band around the critical pressure. At this point, the code has determined all of the necessary pressure points (as shown in Figure 4) and their corresponding properties for the saturation lines. Properties for the critical point are still unobtainable because most property packages, including GASPAK, have problems around the critical point; the property packages either crash or return unreasonable values. Also, constant pressure specific heat ( $C_p$ ) and specific heat ratio ( $\gamma$ ) are infinite at the critical point, and huge values near the critical point are undesirable in the property tables because extremely large values in the tables create numerical computation problems. Therefore,  $C_p$  and  $\gamma$  table values must be of limited magnitude and a different method must be used to acquire the properties near the critical point. This method will be discussed later.

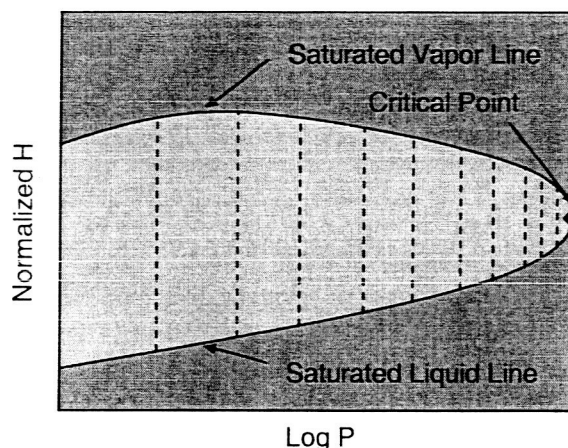


Figure 4. Pressure Grid for Saturation Lines

#### Pressure-Enthalpy Values for Supercritical Region

Next, the program determines pressure-enthalpy values for the supercritical region. This is a more involved process than for the saturation lines, but the methodology remains functionally similar. As stated previously, GASPAK and many other property packages have problems around the critical point; therefore, a pressure band around the critical point is determined by the user to prevent GASPAK from crashing or returning unreasonable values. The pressure band is shown in Figure 5, where  $P_{lo\_crit}$  is the lower end of the band around the critical point, and  $P_{hi\_crit}$  is the higher end. For the supercritical region, the program starts out with the lowest enthalpy ( $H_{min}$ ) and  $P_{hi\_crit}$  and uses those two independent variables to get the first set of property data (shaded circle) for the supercritical region. Next, the program uses the same methodology as it did for the saturation lines to find the second pressure point and its properties (shaded square). Then it continues to march down that enthalpy line and determine the necessary number of pressure points so that interpolation errors along the  $H_{min}$  enthalpy line will be within the specified error.

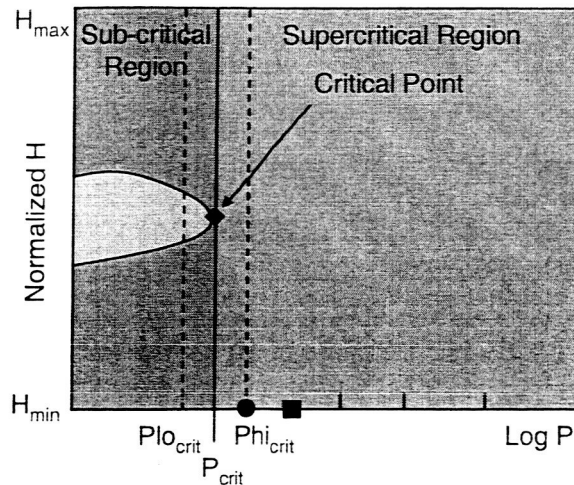


Figure 5. Pressure Band around the Critical Point

After the program finishes with the  $H_{min}$  enthalpy line, it comes back to the  $H_{min}$  and  $\Phi_{crit}$  point, but this time instead of marching down the  $H_{min}$  enthalpy line, it moves along the  $\Phi_{crit}$  pressure line and uses the same methodology with  $\Delta H$  steps in place of  $\Delta P$  steps to determine the enthalpy grid points to fulfill the specified interpolation error limit. The second enthalpy point and its properties for the  $\Phi_{crit}$  pressure line are shown as a shaded triangle in Figure 6. Now, the code moves along the second enthalpy line ( $H_2$ ) and determines pressure points as it did for the  $H_{min}$  enthalpy line. It is the nature of the property data that in the vicinity of the critical point the grids get finer (more data points are needed), so it is expected that the  $H_2$  enthalpy line should have at least the same number or more pressure points than the  $H_{min}$  enthalpy line. Now, the code compares the number of pressure points from the two enthalpy lines. If the  $H_2$  enthalpy line has fewer pressure points than the  $H_{min}$  enthalpy line then the code discards all of the pressure points from the  $H_2$  enthalpy line and only keeps the pressure points from the  $H_{min}$  enthalpy line because now the maximum error criteria along the  $H_2$  enthalpy line will definitely be satisfied by having more pressure points than necessary. Similarly, the code will discard all of the pressure points from the  $H_{min}$  enthalpy line if it has fewer pressure points than the  $H_2$  enthalpy line. Finally if the two enthalpy lines have the same number of pressure points, but with different values, then the code starts over and finds a new set of pressure points that will satisfy both enthalpy lines. To do this, the code starts with the  $\Phi_{crit}$  and increments the pressure by a  $\Delta P$ , as it did before, but now instead of just checking the errors along one enthalpy line, it checks the errors along both enthalpy lines to make sure that all errors are still within the specified limit. So, the next pressure point is determined by the enthalpy line that reaches the specified error first, and the code repeats this process until it finds a new set of pressure points that satisfy both enthalpy lines. This whole methodology is repeated up and across the entire supercritical region until the grid is generated, as shown in Figure 7. In addition to finding the grid for the supercritical region, the code also tries to determine the critical enthalpy based on this grid. As stated previously, constant pressure specific heat is infinite at the critical point. So based on this fact, the code marches down the  $\Phi_{crit}$  pressure line, which is closest to the critical pressure, and selects the enthalpy that has the highest  $C_p$  value, and that enthalpy is defined as the critical enthalpy.

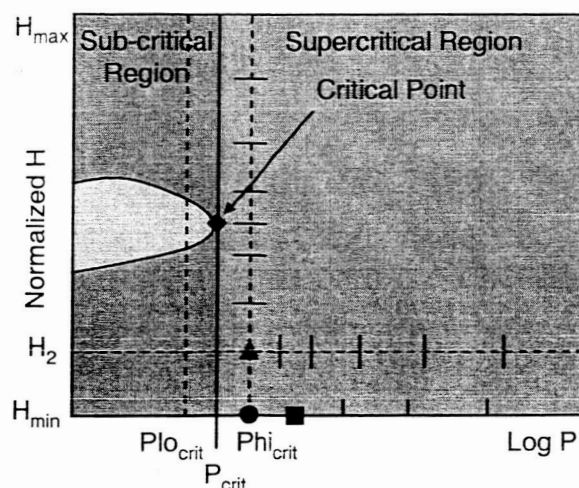


Figure 6. Pressures along  $H_{\min}$ -Enthalpy Line & Enthalpies along  $\Phi_{i,crit}$ -Pressure Line

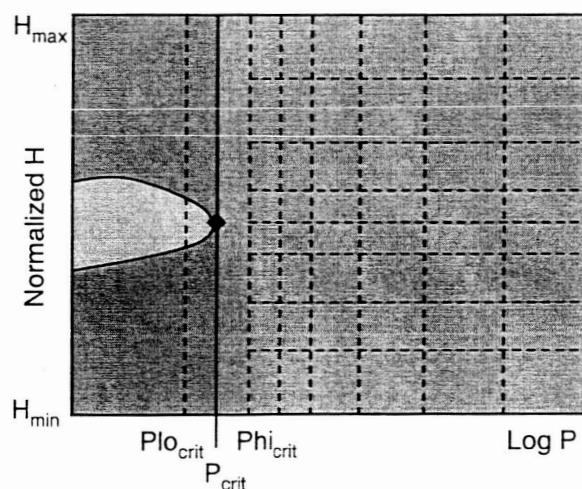


Figure 7. Pressure-Enthalpy Grid for the Supercritical Region

#### Pressure-Enthalpy Values for Sub-critical Region

As shown in Figure 8, the saturation lines lie inside the sub-critical region; therefore, the sub-critical region could have two sets of pressure values. One is for the saturation lines (already determined), and the other is for everything outside of the saturation lines. However, interpolation is more efficient on a grid with only one set of pressure points. So for the sub-critical region, the grid pressure values are the pressure values of the saturation lines; by doing this, the specified error limit is still being satisfied because as mentioned earlier, the grids get finer in the vicinity of the critical point. Hence, if those pressure values satisfy the saturation lines then they will definitely satisfy the rest of the sub-critical region. One could use the same technique that was used to determine the grid for the supercritical region to obtain the enthalpy values for the sub-critical region, and one would find that the sub-critical enthalpies are almost identical to the supercritical enthalpies. So for the sub-critical region, it is acceptable to use the same pressure values as the saturation lines, and the same enthalpy values as the supercritical region. The entire grid, both sub-critical and supercritical, for the properties is shown in Figure 8.

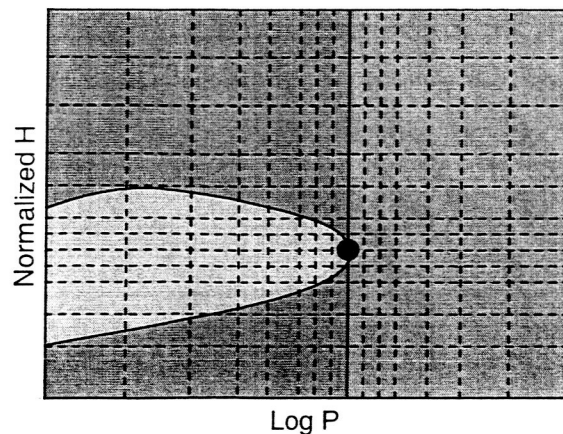


Figure 8. Pressure-Enthalpy Grid for the Entire Region

### Generation of Property Tables

There are two property tables for each thermodynamic property. The first table is the saturation table (Property Table 1), which includes the saturated liquid and saturated vapor properties, and the second table (Property Table 2) contains everything but saturated properties. To generate property data for Property Table 1, the program uses previously determined saturated pressures as shown in Figure 4 as inputs into GASPAK and selects an option to return saturated properties. On the other hand, the program uses previously determined pressure-enthalpy pairs (two independent properties) as shown in Figure 8 to obtain property data for Property Table 2. However, the program does not generate any property data for those pressure-enthalpy pairs that are inside the saturation envelope because GASPAK returns zero values for most of those thermodynamic properties. The program simply puts zeros for all of those data points into Property Table 2.

As mentioned earlier, there is a pressure band around the critical point to prevent GASPAK from crashing or returning unreasonable values. There is also an enthalpy band around the critical point for the same reason. For those pressure-enthalpy pairs that are inside the critical point pressure-enthalpy band, the program does not use GASPAK to generate data; instead, it uses the linear interpolation method described by Equation 1 to get the property data. For example to get the property data for the critical point (as shown in Figure 9), which is inside the pressure-enthalpy band, the program interpolates between the nearest point below the lo-enthalpy line (shaded triangle) and the nearest point above the hi-enthalpy line (shaded square) using the critical enthalpy ( $H_{crit}$ ) as input. Now for properties which have maximums at the critical point, such as constant volume specific heat ( $C_v$ ), constant pressure specific heat ( $C_p$ ), specific heat ratio ( $\gamma$ ), and thermal conductivity ( $k$ ), linear interpolation is not used. Instead, the program multiplies the higher value of the two data points by a constant that is greater than one. For the speed of sound ( $a$ ), which is at a minimum at the critical point, the program multiplies the lower value by a constant that is smaller than one. These constants are user selectable to allow for adjustment if numerical problems are encountered.

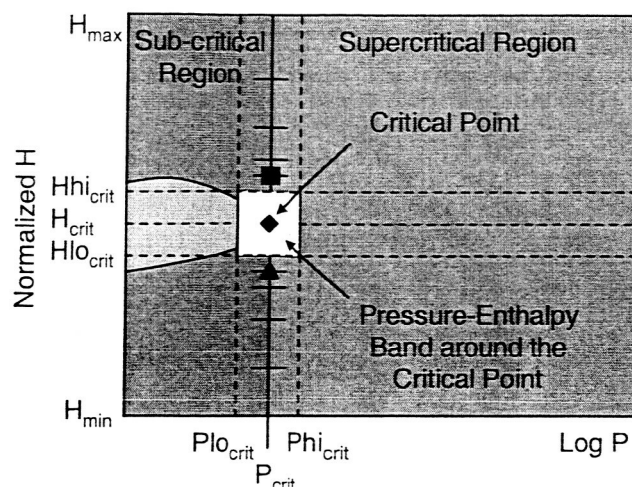


Figure 9. Pressure-Enthalpy Band around the Critical Point

The aforementioned program could be used to determine separate grids for each fluid property one at a time or to determine one grid for multiple fluid properties at the same time. In other words, one could use the program to obtain a grid for density and then use the program again to obtain a grid for specific heat, and most likely, the two grids would not be the same. On the other hand, one could use the program to determine only one grid that would satisfy the specified interpolation error limits for both density and specific heat. Figures 10 through 14 demonstrate what an oxygen grid defined for all properties at one time would look like. As expected, the grid gets finer in the vicinity of the critical point. The figures show how oxygen properties behave across the entire range of the tables. The program does not generate any property data for those pressure-enthalpy pairs that are inside the dome because GASPAC returns zero values for most of the thermodynamic properties, so the program simply puts zeros for all of those data points.

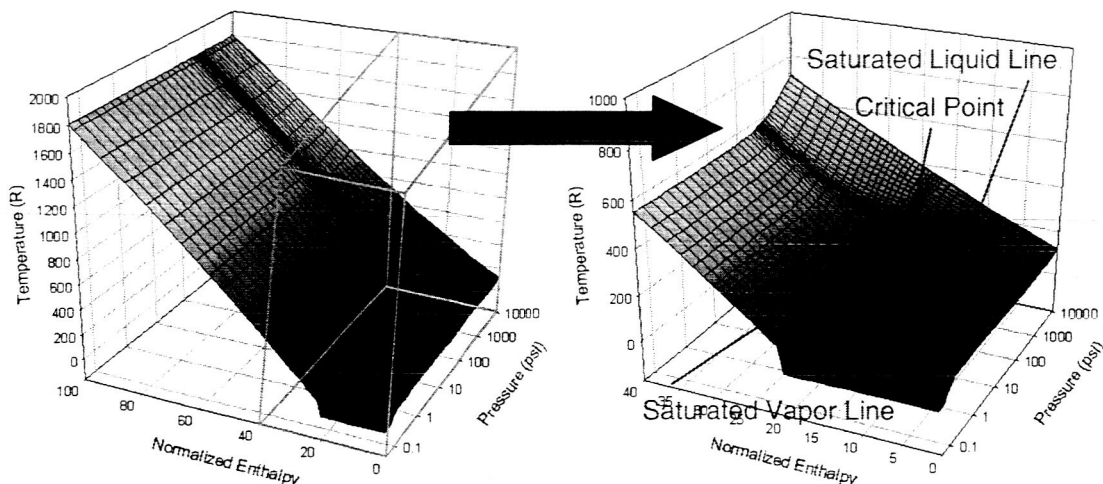


Figure 10. Oxygen Temperature

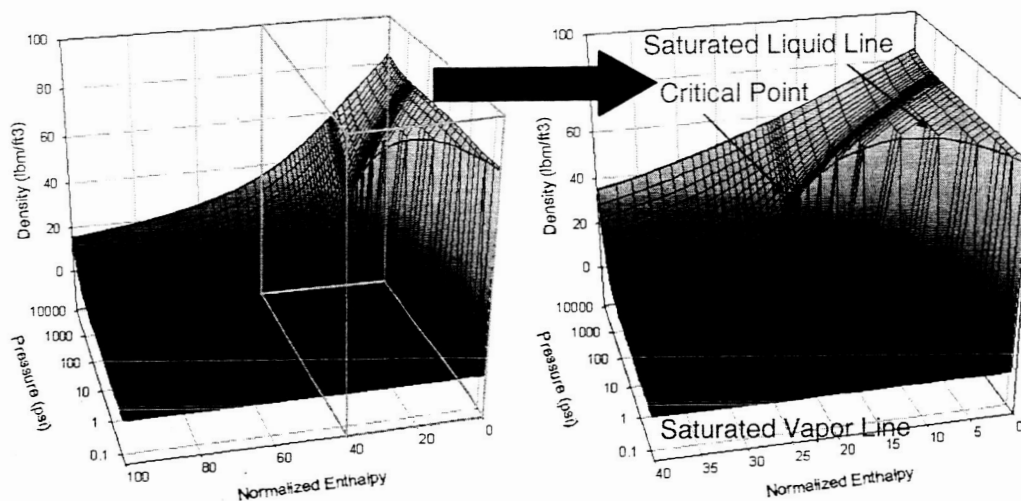


Figure 11. Oxygen Density

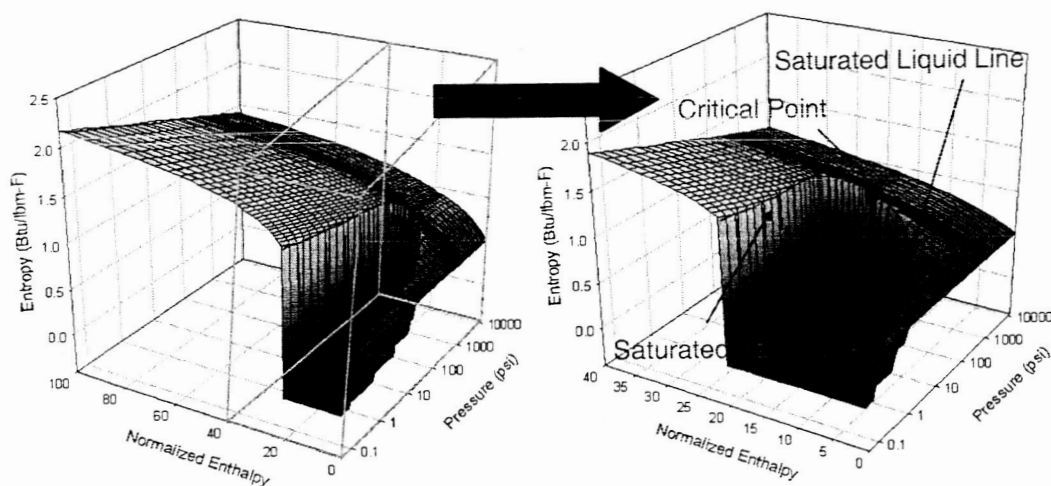


Figure 12. Oxygen Entropy

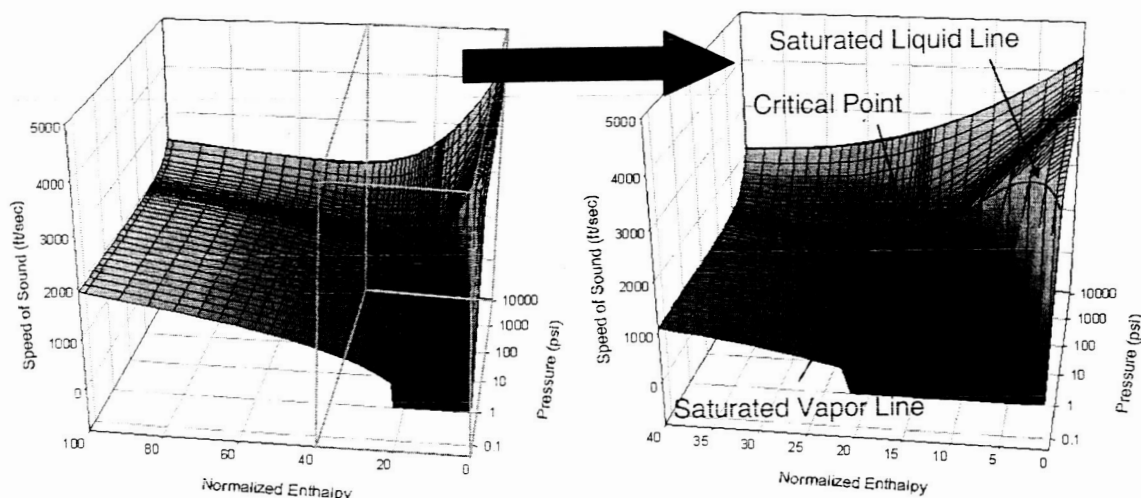


Figure 13. Oxygen Speed of Sound

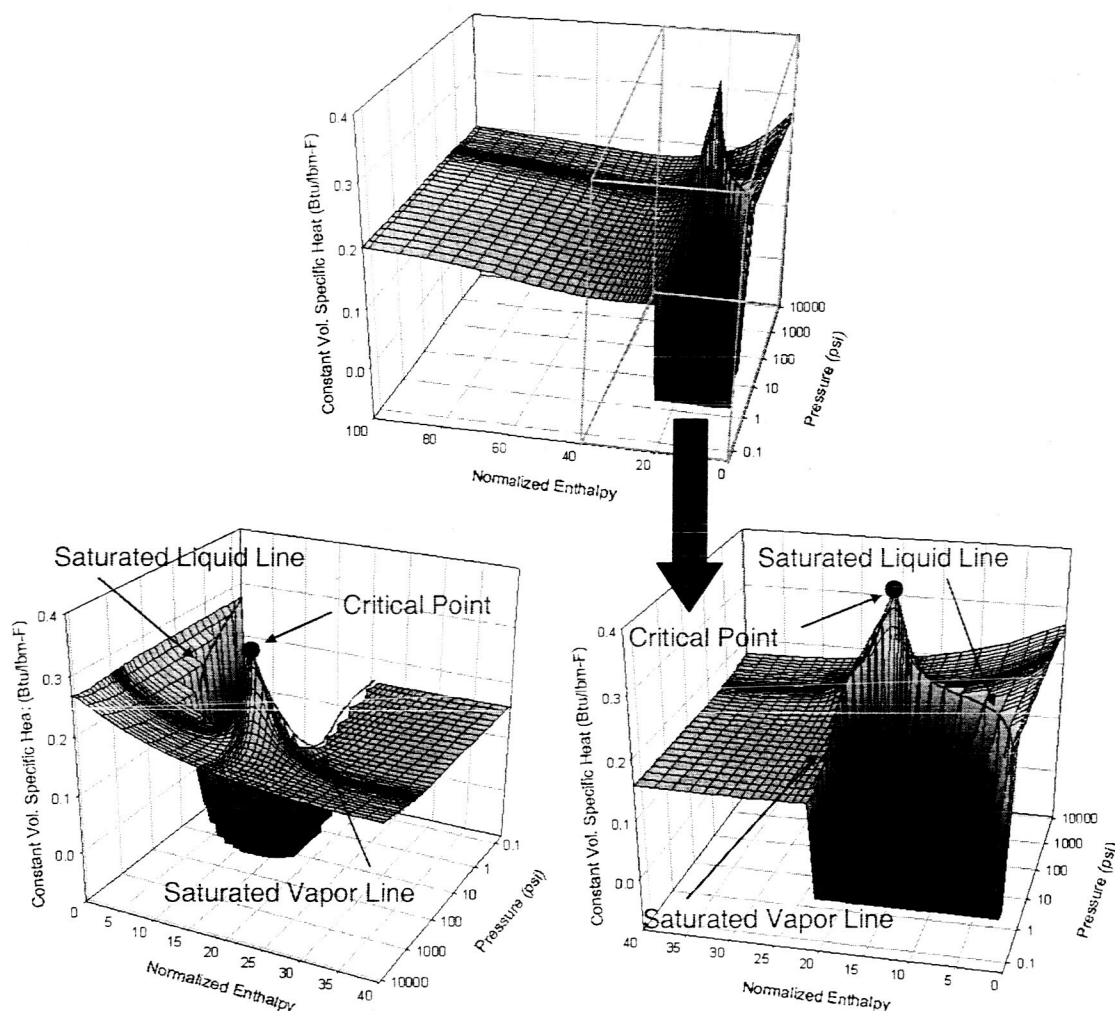


Figure 14. Oxygen Constant Volume Specific Heat

## INTERPOLATION METHODS

After having evaluated the state space for a given fluid property, the data is compiled into tables. Two tables were generated for each thermodynamic property. The first table (Property Table 1) only includes saturated vapor and saturated liquid properties, while the second table (Property Table 2) contains everything but saturated properties. Then, a means of utilizing these tables in an accurate manner across the full state space was developed. For cases within the saturation envelope (or dome), data along the saturation lines (Property Table 1) are used along with fluid quality to interpolate for composite fluid properties. For the supercritical region and much of the sub-critical region, the table-reading method used is straightforward interpolation of Property Table 2. However, in the sub-critical region near the saturation lines, a special interpolation technique is required where one or more data points, from Property Table 2, needed for interpolation falls inside the saturation dome. Figure 15 shows fourteen possible cases near the saturation lines where a special interpolation method is required in order to retrieve accurate property data from the tables.

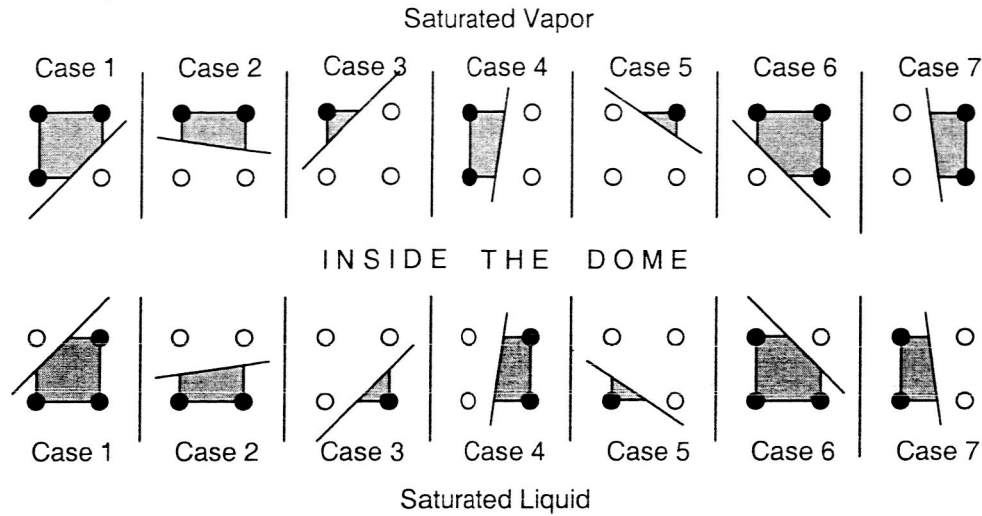


Figure 15. Possible Special Interpolation Cases near Saturation Lines

#### Inside the Saturation Envelope

For those cases that are inside the saturation envelope, the properties can be calculated by using quality ( $X$ ) and the saturation data. Quality is the ratio of the mass of vapor to the total mass of the mixture, and it can be expressed mathematically as

$$X = \frac{m_v}{m_L + m_v} \quad \text{Equation (3)}$$

where  $m_v$  is the vapor mass, and  $m_L$  is the liquid mass. Equation 3 shows that if there is no vapor mass then the quality is 0, but on the other hand, if there is no liquid mass then the quality is 1. So, it is obvious that quality must be between 0 and 1. Figure 16 shows an example of a case inside the saturation envelope and illustrates how one can retrieve property data from Property Table 1 using quality. The first step in determining properties inside the saturation envelope is to obtain saturated liquid (shaded square in Figure 16) and saturated vapor (unshaded square) properties from Property Table 1 by using the input pressure ( $P$ ) as input, and the next step is to calculate quality by using Equation 4 (which can easily be found in any thermodynamic book)

$$X = \frac{H - H_L}{H_V - H_L} \quad \text{Equation (4)}$$

where  $H$  is the input enthalpy,  $H_L$  is the saturated liquid enthalpy, and  $H_V$  is the saturated vapor enthalpy. Now by rearranging Equation (4) and replacing enthalpy with a generic property called  $Z$ , Equation (4) becomes

$$Z = Z_L + X(Z_V - Z_L) \quad \text{Equation (5)}$$

and this equation is used to determine any mixture property, except for mixture density, that is inside the dome by replacing  $Z_L$  and  $Z_V$  with the appropriate property data. For density, the generic property,  $Z$ , is replaced with specific volume ( $V$ ) and by using the fact that specific volume is the inverse of density, Equation (5) is rewritten as

$$\frac{1}{\rho} = \frac{1}{\rho_L} + X \left( \frac{1}{\rho_V} - \frac{1}{\rho_L} \right) \rightarrow \rho = \frac{1}{\frac{1}{\rho_L} + X \left( \frac{1}{\rho_V} - \frac{1}{\rho_L} \right)}$$

Equation (6)

and the above equation is used to calculate mixture density.

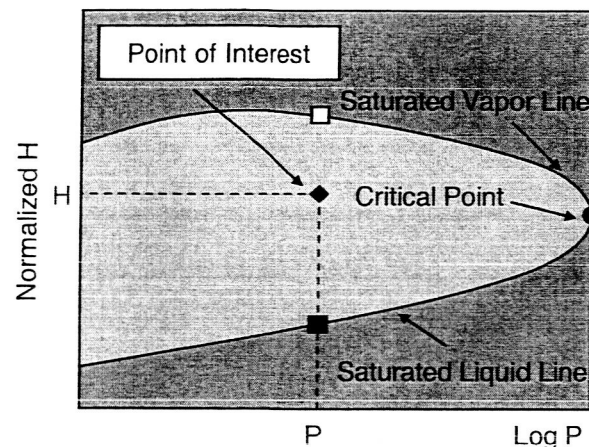


Figure 16. A Case inside the Saturation Envelope

#### Non-Special Interpolation Cases

All cases of the supercritical region and most of the cases in the sub-critical region do not require the use of the special interpolation technique to retrieve property data from the property tables; instead, the method used is straightforward interpolation. Figure 17 shows an example of a case in the supercritical region, and it illustrates how one can retrieve property data from Property Table 2 using straightforward linear interpolation described by Equation 1. The interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the properties along the  $H_{lo}$  enthalpy line (shaded circle) can be determined by interpolating between the bottom two data points from Property Table 2 using the input pressure  $P$  as the independent variable ( $x$ ) in Equation 1. In the same way, the properties along the  $H_{hi}$  enthalpy line (shaded square) are computed by interpolating between the top two data points from Property Table 2 using the same input pressure. Finally, the properties of interest are obtained by interpolating between the two newfound sets of properties using the input enthalpy  $H$  as the independent variable ( $x$ ) in Equation 1.

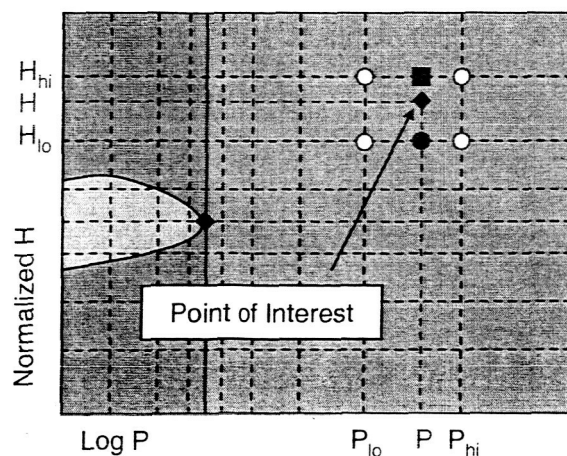


Figure 17. Non-Special Interpolation Case in the Supercritical Region

### Special Interpolation Cases

Only fourteen cases (as shown in Figure 15) require the use of the special interpolation technique to retrieve property data from the property tables, and these cases are where one or more needed grid points from Property Table 2 fall inside the saturation envelope. An examination of Figure 15 reveals that many of the cases are mirror images of other cases. For example, Saturated Vapor Case 2 is a mirror image of Saturated Liquid Case 2, and Saturated Vapor Case 4 is a mirror image of Saturated Vapor Case 7.

### Saturated Liquid Case 1

Saturated Liquid Case 1 as defined in Figure 15 is shown in Figure 18 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the top left data point (at  $P_{lo}$  and  $H_{hi}$ ) can not be used in the interpolation process because it lies inside the saturation envelope. First, it is necessary to determine the pressure ( $P_{H_{hi}}$ ) at which the saturated liquid line crosses the higher enthalpy line ( $H_{hi}$ ) of the interpolation rectangle. The first step in determining  $P_{H_{hi}}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{P_{lo}}$ ) and  $P_{hi}$  ( $H_{P_{hi}}$ ) using Property Table 1; then by using Equation 1,  $P_{H_{hi}}$  can be calculated by interpolating between  $H_{P_{lo}}$  and  $H_{P_{hi}}$  using  $H_{hi}$  as input. Next, the input pressure ( $P$ ) is compared to  $P_{H_{hi}}$ , and the comparison defines two sub-cases within Saturated Liquid Case 1. One sub-case is where  $P$  is less than  $P_{H_{hi}}$ , and the other sub-case is where  $P$  is greater than or equal to  $P_{H_{hi}}$ .

### *Sub-case 1*

The first sub-case, where  $P$  is less than  $P_{H_{hi}}$ , is shown in Figure 18. The saturated properties for  $P$  (shaded triangle) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using the input pressure  $P$ . Likewise, the properties for  $H_{lo}$  (shaded circle) are obtained by interpolating between the bottom two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{lo}$ ) from Property Table 2 using  $P$  as input. Finally, the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy ( $H$ ).

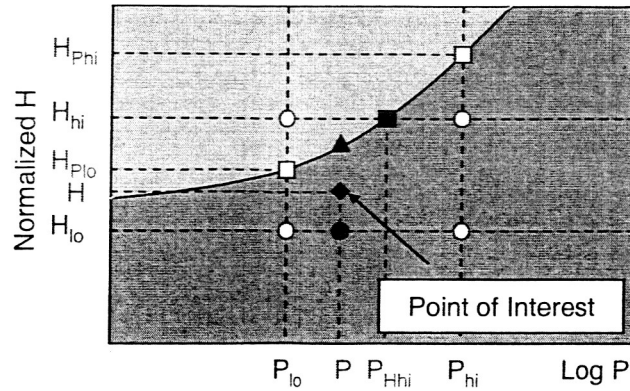


Figure 18. Saturated Liquid Case 1 – Sub-case 1

### Sub-case 2

The second sub-case, where  $P$  is greater than or equal to  $P_{Hhi}$ , is shown in Figure 19. The saturated properties for  $P_{Hhi}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hhi}$  as input. Now the properties for  $H_{hi}$  (shaded triangle) are calculated by interpolating between the  $P_{Hhi}$  saturated properties (shaded square) and the top right data point (at  $P_{hi}$  and  $H_{hi}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the properties for  $H_{lo}$  (shaded circle) are obtained by interpolating between the bottom two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{lo}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded circle and shaded triangle) using the input enthalpy  $H$ .

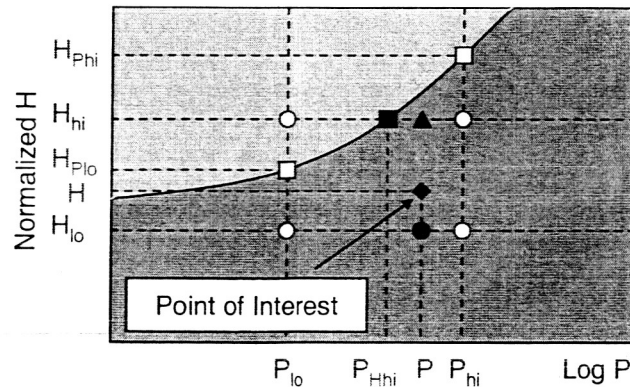


Figure 19. Saturated Liquid Case 1 – Sub-case 2

### Saturated Liquid Case 2

Saturated Liquid Case 2 as defined in Figure 15 is shown in Figure 20 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the top two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{hi}$ ) can not be used in the interpolation process because they lie inside the saturation envelope. To interpolate for the point of interest, the bottom two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{lo}$ ) from Property Table 2 are used along with two other data points from Property Table 1. Now the saturated properties for  $P$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using the input pressure ( $P$ ). Likewise, the properties for  $H_{lo}$  (shaded circle) are obtained by interpolating between the bottom two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{lo}$ ) from Property Table 2 using

P as input. Finally, the properties of interest at P and H (shaded diamond) are determined by interpolating between the  $H_{lo}$  properties (shaded circle) and the saturated properties for P (shaded square) using the input enthalpy (H).

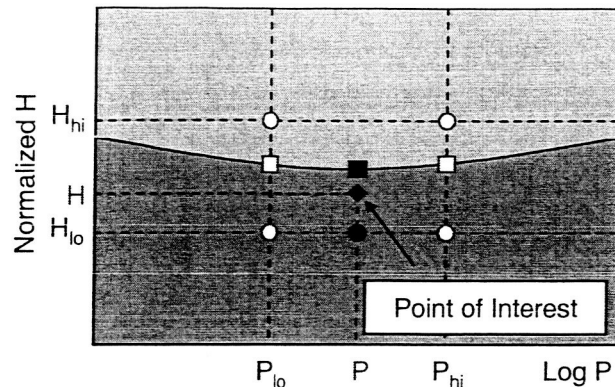


Figure 20. Saturated Liquid Case 2

### Saturated Liquid Case 3

Saturated Liquid Case 3 as defined in Figure 15 is shown in Figure 21 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and three of the four data points can not be used in the interpolation process because they lie inside the saturation envelope. To interpolate for the point of interest, the bottom right data point (at  $P_{hi}$  and  $H_{lo}$ ) from Property Table 2 is used along with two other data points from Property Table 1. First, it is necessary to determine the pressure ( $P_{Hlo}$ ) at which the saturated liquid line crosses the lower enthalpy line ( $H_{lo}$ ) of the interpolation rectangle. The first step in determining  $P_{Hlo}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{Plo}$ ) and  $P_{hi}$  ( $H_{Phi}$ ) using Property Table 1; then by using Equation 1,  $P_{Hlo}$  can be calculated by interpolating between  $H_{Plo}$  and  $H_{Phi}$  using  $H_{lo}$  as input. Next, the saturated properties for  $P_{Hlo}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hlo}$  as input. Now the properties for  $H_{lo}$  (shaded circle) are calculated by interpolating between the  $P_{Hlo}$  saturated properties (shaded square) and the bottom right data point (at  $P_{hi}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure (P). Similarly, the saturated properties for P (shaded triangle) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using P as input. Finally the properties of interest at P and H (shaded diamond) are determined by interpolating between the  $H_{lo}$  properties (shaded circle) and the saturated properties for P (shaded triangle) using the input enthalpy (H).

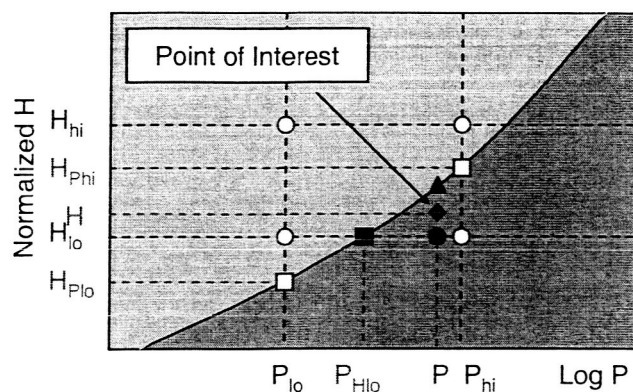


Figure 21. Saturated Liquid Case 3

### Saturated Liquid Case 4

Saturated Liquid Case 4 as defined in Figure 15 is shown in Figure 22 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the two left data points ( $P_{lo}$  at  $H_{lo}$  and at  $H_{hi}$ ) can not be used in the interpolation process because they lie inside the saturation envelope. First, it is necessary to determine the pressure ( $P_{Hhi}$ ) at which the saturated liquid line crosses the higher enthalpy line ( $H_{hi}$ ) of the interpolation rectangle. The first step in determining  $P_{Hhi}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{Plo}$ ) and  $P_{hi}$  ( $H_{Phi}$ ) using Property Table 1; then by using Equation 1,  $P_{Hhi}$  can be calculated by interpolating between  $H_{Plo}$  and  $H_{Phi}$  using  $H_{hi}$  as input. Similarly,  $P_{Hlo}$  is the pressure at which the saturated liquid line crosses the lower enthalpy line ( $H_{lo}$ ) of the interpolation rectangle, and it can be obtained by interpolating between  $H_{Plo}$  and  $H_{Phi}$  using  $H_{lo}$  as input. Next, the input pressure ( $P$ ) is compared to  $P_{Hhi}$ , and the comparison defines two sub-cases within Saturated Liquid Case 4. One sub-case is where  $P$  is less than  $P_{Hhi}$ , and the other sub-case is where  $P$  is greater than or equal to  $P_{Hhi}$ .

#### *Sub-case 1*

The first sub-case, where  $P$  is less than  $P_{Hhi}$ , is shown in Figure 22. The saturated properties for  $P_{Hlo}$  (shaded trapezoid) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hlo}$  as input. Now the properties for  $H_{lo}$  (shaded circle) are calculated by interpolating between the  $P_{Hlo}$  saturated properties (shaded trapezoid) and the bottom right data point (at  $P_{hi}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the saturated properties for  $P$  (shaded triangle) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy ( $H$ ).

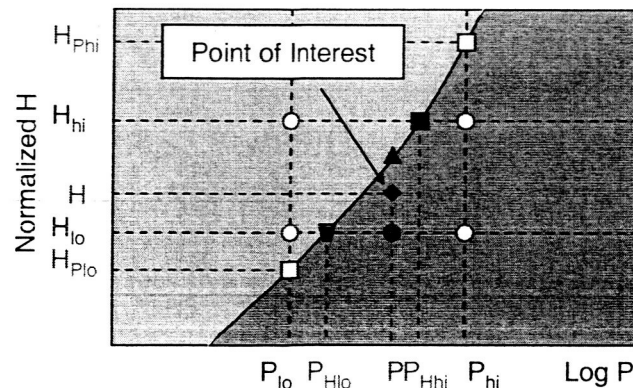


Figure 22. Saturated Liquid Case 4 – Sub-case 1

#### *Sub-case 2*

The second sub-case, where  $P$  is greater than or equal to  $P_{Hhi}$ , is shown in Figure 23. The saturated properties for  $P_{Hhi}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hhi}$  as input. Now the properties for  $H_{hi}$  (shaded triangle) are calculated by interpolating between the  $P_{Hhi}$  saturated properties (shaded square) and the top right data point (at  $P_{hi}$  and  $H_{hi}$ ) from Property Table 2 using the input

pressure  $P$ . Similarly, the properties for  $H_{lo}$  (shaded circle) are obtained by interpolating between the  $P_{Hlo}$  saturated properties (shaded trapezoid) and the bottom right data point (at  $P_{hi}$  and  $H_{lo}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded circle and shaded triangle) using the input enthalpy  $H$ .

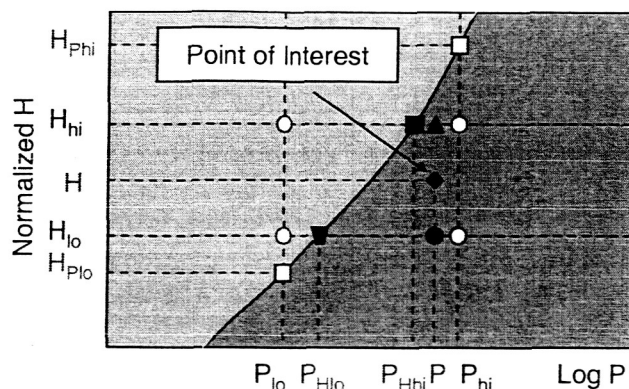


Figure 23. Saturated Liquid Case 4 – Sub-case 2

#### Saturated Liquid Case 5

Saturated Liquid Case 5 as defined in Figure 15 is shown in Figure 24 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and three of the four data points can not be used in the interpolation process because they lie inside the saturation envelope. To interpolate for the point of interest, the bottom left data point (at  $P_{lo}$  and  $H_{lo}$ ) from Property Table 2 is used along with two other data points from Property Table 1. First, it is necessary to determine the pressure ( $P_{Hlo}$ ) at which the saturated liquid line crosses the lower enthalpy line ( $H_{lo}$ ) of the interpolation rectangle. The first step in determining  $P_{Hlo}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{Plo}$ ) and  $P_{hi}$  ( $H_{Phi}$ ) using Property Table 1; then by using Equation 1,  $P_{Hlo}$  can be calculated by interpolating between  $H_{Plo}$  and  $H_{Phi}$  using  $H_{lo}$  as input. Next, the saturated properties for  $P_{Hlo}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hlo}$  as input. Now the properties for  $H_{lo}$  (shaded circle) are calculated by interpolating between the  $P_{Hlo}$  saturated properties (shaded square) and the bottom left data point (at  $P_{lo}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure ( $P$ ). Similarly, the saturated properties for  $P$  (shaded triangle) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy ( $H$ ).

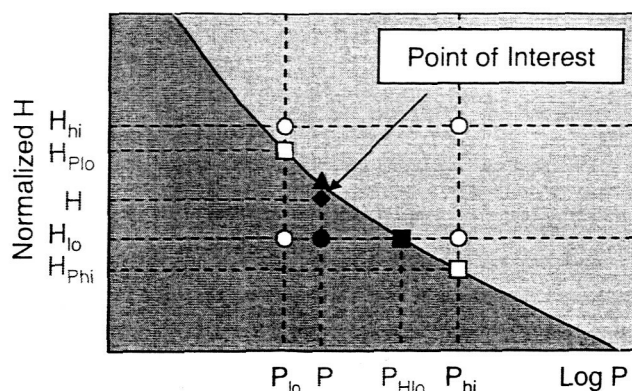


Figure 24. Saturated Liquid Case 5

### Saturated Liquid Case 6

Saturated Liquid Case 6 as defined in Figure 15 is shown in Figure 25 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the top right data point (at  $P_{hi}$  and  $H_{hi}$ ) can not be used in the interpolation process because it lies inside the saturation envelope. First, it is necessary to determine the pressure ( $P_{Hhi}$ ) at which the saturated liquid line crosses the higher enthalpy line ( $H_{hi}$ ) of the interpolation rectangle. The first step in determining  $P_{Hhi}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{Plo}$ ) and  $P_{hi}$  ( $H_{Pphi}$ ) using Property Table 1; then by using Equation 1,  $P_{Hhi}$  can be calculated by interpolating between  $H_{Plo}$  and  $H_{Pphi}$  using  $H_{hi}$  as input. Next, the input pressure ( $P$ ) is compared to  $P_{Hhi}$ , and the comparison defines two sub-cases within Saturated Liquid Case 6. One sub-case is where  $P$  is less than  $P_{Hhi}$ , and the other sub-case is where  $P$  is greater than or equal to  $P_{Hhi}$ .

#### *Sub-case 1*

The first sub-case, where  $P$  is less than  $P_{Hhi}$ , is shown in Figure 25. The saturated properties for  $P_{Hhi}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hhi}$  as input. Now the properties for  $H_{hi}$  (shaded triangle) are calculated by interpolating between the  $P_{Hhi}$  saturated properties (shaded square) and the top left data point (at  $P_{lo}$  and  $H_{hi}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the properties for  $H_{lo}$  (shaded circle) are obtained by interpolating between the bottom two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{lo}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded circle and shaded triangle) using the input enthalpy ( $H$ ).

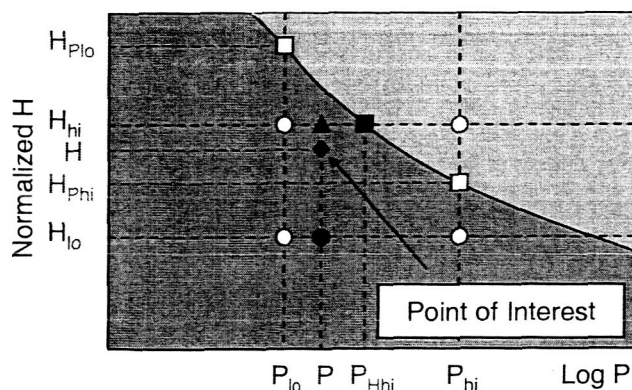


Figure 25. Saturated Liquid Case 6 – Sub-case 1

### Sub-case 2

The second sub-case, where  $P$  is greater than or equal to  $P_{Hhi}$ , is shown in Figure 26. The saturated properties for  $P$  (shaded triangle) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using the input pressure  $P$ . Likewise, the properties for  $H_{lo}$  (shaded circle) are obtained by interpolating between the bottom two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{lo}$ ) from Property Table 2 using  $P$  as input. Finally, the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy  $H$ .

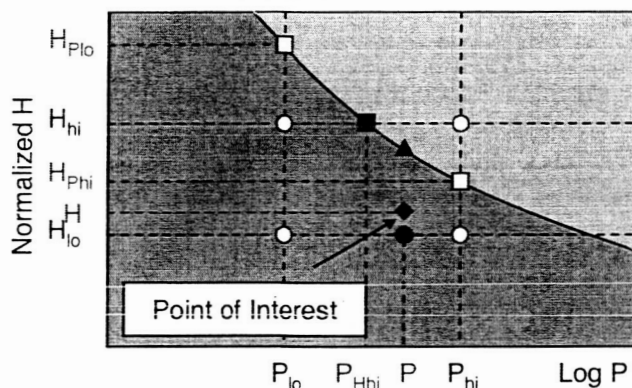


Figure 26. Saturated Liquid Case 6 – Sub-case 2

### Saturated Liquid Case 7

Saturated Liquid Case 7 as defined in Figure 15 is shown in Figure 27 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the two right data points ( $P_{hi}$  at  $H_{lo}$  and at  $H_{hi}$ ) can not be used in the interpolation process because they lie inside the saturation envelope. First, it is necessary to determine the pressure ( $P_{Hhi}$ ) at which the saturated liquid line crosses the higher enthalpy line ( $H_{hi}$ ) of the interpolation rectangle. The first step in determining  $P_{Hhi}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{P_{lo}}$ ) and  $P_{hi}$  ( $H_{P_{hi}}$ ) using Property Table 1; then by using Equation 1,  $P_{Hhi}$  can be calculated by interpolating between  $H_{P_{lo}}$  and  $H_{P_{hi}}$  using  $H_{hi}$  as input. Similarly,  $P_{Hlo}$  is the pressure at which the saturated liquid line crosses the lower enthalpy line ( $H_{lo}$ ) of the interpolation rectangle, and it can be obtained by interpolating between  $H_{P_{lo}}$  and  $H_{P_{hi}}$  using  $H_{lo}$  as input. Next, the input pressure ( $P$ ) is compared to  $P_{Hhi}$ , and the comparison defines two sub-cases within Saturated Liquid Case 7. One sub-case is where  $P$  is less than  $P_{Hhi}$ , and the other sub-case is where  $P$  is greater than or equal to  $P_{Hhi}$ .

### Sub-case 1

The first sub-case, where  $P$  is less than  $P_{Hhi}$ , is shown in Figure 27. The saturated properties for  $P_{Hhi}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hhi}$  as input. Now the properties for  $H_{hi}$  (shaded triangle) are calculated by interpolating between the  $P_{Hhi}$  saturated properties (shaded square) and the top left data point (at  $P_{lo}$  and  $H_{hi}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the saturated properties for  $P_{Hlo}$  (shaded trapezoid) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hlo}$  as input. Next the

properties for  $H_{lo}$  (shaded circle) are computed by interpolating between the  $P_{Hlo}$  saturated properties (shaded trapezoid) and the bottom left data point (at  $P_{lo}$  and  $H_{lo}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded circle and shaded triangle) using the input enthalpy ( $H$ ).

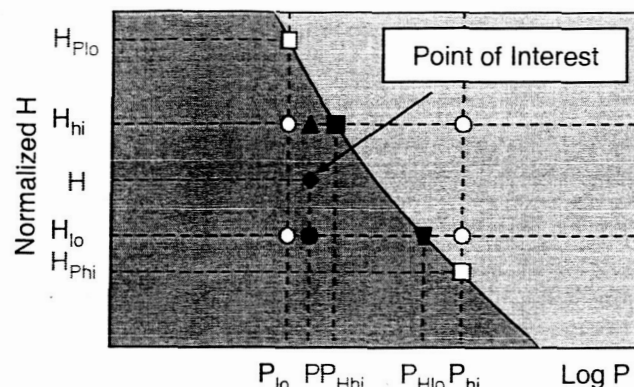


Figure 27. Saturated Liquid Case 7 – Sub-case 1

#### *Sub-case 2*

The second sub-case, where  $P$  is greater than or equal to  $P_{Hhi}$ , is shown in Figure 28. The properties for  $H_{lo}$  (shaded circle) are calculated by interpolating between the  $P_{Hlo}$  saturated properties (shaded trapezoid) and the bottom left data point (at  $P_{lo}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the saturated properties for  $P$  (shaded triangle) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy  $H$ .

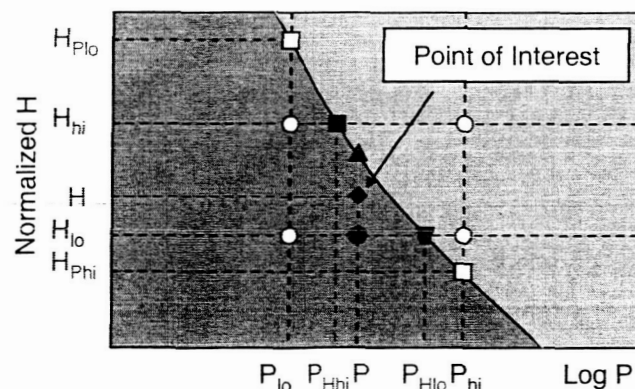


Figure 28. Saturated Liquid Case 7 – Sub-case 2

#### Saturated Vapor Case 1

Saturated Vapor Case 1 as defined in Figure 15 is shown in Figure 29 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the bottom right data point (at  $P_{hi}$  and  $H_{lo}$ ) can not be used in the interpolation process because it lies inside the saturation envelope. First, it is necessary to determine the pressure ( $P_{Hlo}$ ) at which the saturated vapor line crosses the lower enthalpy line ( $H_{lo}$ ) of the

interpolation rectangle. The first step in determining  $P_{H_{lo}}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{P_{lo}}$ ) and  $P_{hi}$  ( $H_{P_{hi}}$ ) using Property Table 1; then by using Equation 1,  $P_{H_{lo}}$  can be calculated by interpolating between  $H_{P_{lo}}$  and  $H_{P_{hi}}$  using  $H_{lo}$  as input. Next, the input pressure ( $P$ ) is compared to  $P_{H_{lo}}$ , and the comparison defines two sub-cases within Saturated Vapor Case 1. One sub-case is where  $P$  is less than  $P_{H_{lo}}$ , and the other sub-case is where  $P$  is greater than or equal to  $P_{H_{lo}}$ .

### Sub-case 1

The first sub-case, where  $P$  is less than  $P_{H_{lo}}$ , is shown in Figure 29. The saturated properties for  $P_{H_{lo}}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{H_{lo}}$  as input. Now the properties for  $H_{lo}$  (shaded triangle) are calculated by interpolating between the  $P_{H_{lo}}$  saturated properties (shaded square) and the bottom left data point (at  $P_{lo}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the properties for  $H_{hi}$  (shaded circle) are obtained by interpolating between the top two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{hi}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded triangle and shaded circle) using the input enthalpy ( $H$ ).

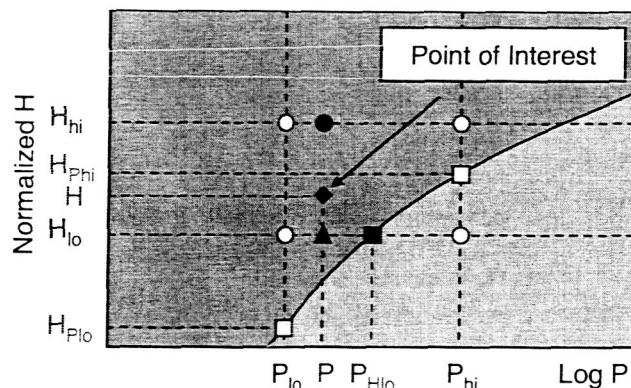


Figure 29. Saturated Vapor Case 1 – Sub-case 1

### Sub-case 2

The second sub-case, where  $P$  is greater than or equal to  $P_{H_{lo}}$ , is shown in Figure 30. The saturated properties for  $P$  (shaded triangle) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using the input pressure  $P$ . Likewise, the properties for  $H_{hi}$  (shaded circle) are obtained by interpolating between the top two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{hi}$ ) from Property Table 2 using  $P$  as input. Finally, the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{hi}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy  $H$ .

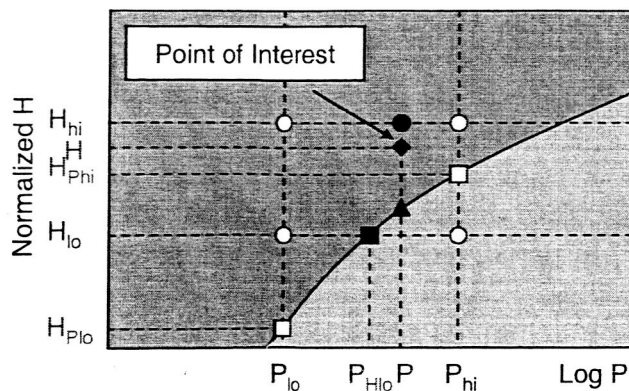


Figure 30. Saturated Vapor Case 1 – Sub-case 2

### Saturated Vapor Case 2

Saturated Vapor Case 2 as defined in Figure 15 is shown in Figure 31 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the bottom two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{lo}$ ) can not be used in the interpolation process because they lie inside the saturation envelope. To interpolate for the point of interest, the top two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{hi}$ ) from Property Table 2 are used along with two other data points from Property Table 1. Now the saturated properties for  $P$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using the input pressure ( $P$ ). Likewise, the properties for  $H_{hi}$  (shaded circle) are obtained by interpolating between the top two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{hi}$ ) from Property Table 2 using  $P$  as input. Finally, the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{hi}$  properties (shaded circle) and the saturated properties for  $P$  (shaded square) using the input enthalpy ( $H$ ).

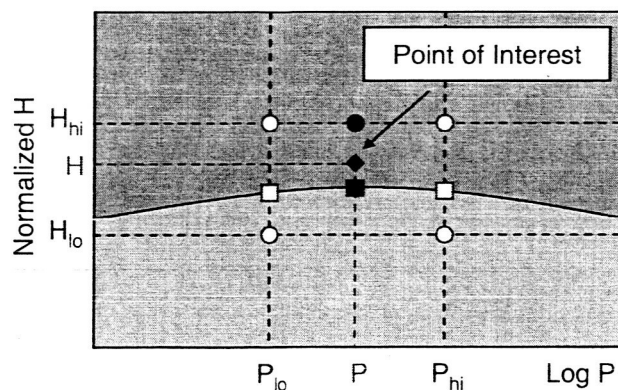


Figure 31. Saturated Vapor Case 2

### Saturated Vapor Case 3

Saturated Vapor Case 3 as defined in Figure 15 is shown in Figure 32 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and three of the four data points can not be used in the interpolation process because they lie inside the saturation envelope. To interpolate for the point of interest, the top left data point (at  $P_{lo}$  and  $H_{hi}$ ) from Property Table 2 is used along with two other data points from Property Table 1. First, it is necessary to determine the pressure ( $P_{Hhi}$ ) at which the saturated vapor line crosses the higher enthalpy line ( $H_{hi}$ ) of the interpolation rectangle. The first step in

determining  $P_{Hhi}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{Plo}$ ) and  $P_{hi}$  ( $H_{Phi}$ ) using Property Table 1; then by using Equation 1,  $P_{Hhi}$  can be calculated by interpolating between  $H_{Plo}$  and  $H_{Phi}$  using  $H_{hi}$  as input. Next, the saturated properties for  $P_{Hhi}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hhi}$  as input. Now the properties for  $H_{hi}$  (shaded circle) are calculated by interpolating between the  $P_{Hhi}$  saturated properties (shaded square) and the top left data point (at  $P_{lo}$  and  $H_{hi}$ ) from Property Table 2 using the input pressure ( $P$ ). Similarly, the saturated properties for  $P$  (shaded triangle) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{hi}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy ( $H$ ).

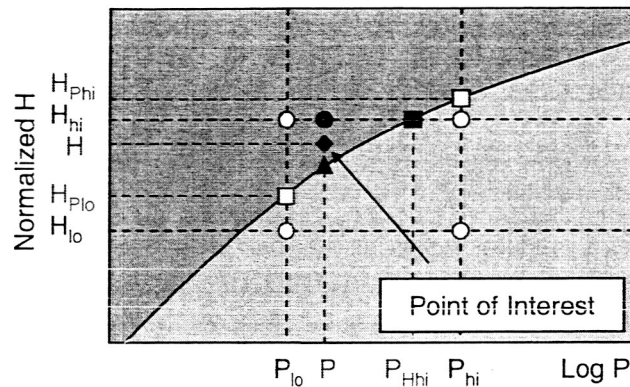


Figure 32. Saturated Vapor Case 3

#### Saturated Vapor Case 4

Saturated Vapor Case 4 as defined in Figure 15 is shown in Figure 33 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the two right data points ( $P_{hi}$  at  $H_{lo}$  and at  $H_{hi}$ ) can not be used in the interpolation process because they lie inside the saturation envelope. First, it is necessary to determine the pressure ( $P_{Hlo}$ ) at which the saturated vapor line crosses the lower enthalpy line ( $H_{lo}$ ) of the interpolation rectangle. The first step in determining  $P_{Hlo}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{Plo}$ ) and  $P_{hi}$  ( $H_{Phi}$ ) using Property Table 1; then by using Equation 1,  $P_{Hlo}$  can be calculated by interpolating between  $H_{Plo}$  and  $H_{Phi}$  using  $H_{lo}$  as input. Similarly,  $P_{Hhi}$  is the pressure at which the saturated vapor line crosses the higher enthalpy line ( $H_{hi}$ ) of the interpolation rectangle, and it can be obtained by interpolating between  $H_{Plo}$  and  $H_{Phi}$  using  $H_{hi}$  as input. Next, the input pressure ( $P$ ) is compared to  $P_{Hlo}$ , and the comparison defines two sub-cases within Saturated Vapor Case 4. One sub-case is where  $P$  is less than  $P_{Hlo}$ , and the other sub-case is where  $P$  is greater than or equal to  $P_{Hlo}$ .

#### *Sub-case 1*

The first sub-case, where  $P$  is less than  $P_{Hlo}$ , is shown in Figure 33. The saturated properties for  $P_{Hlo}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hlo}$  as input. Now the properties for  $H_{lo}$  (shaded triangle) are calculated by interpolating between the  $P_{Hlo}$  saturated properties (shaded square) and the bottom left data point (at  $P_{lo}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the saturated properties for  $P_{Hhi}$  (shaded trapezoid) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hhi}$  as input. Next the

properties for  $H_{hi}$  (shaded circle) are computed by interpolating between the  $P_{Hhi}$  saturated properties (shaded trapezoid) and the top left data point (at  $P_{lo}$  and  $H_{hi}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded triangle and shaded circle) using the input enthalpy ( $H$ ).

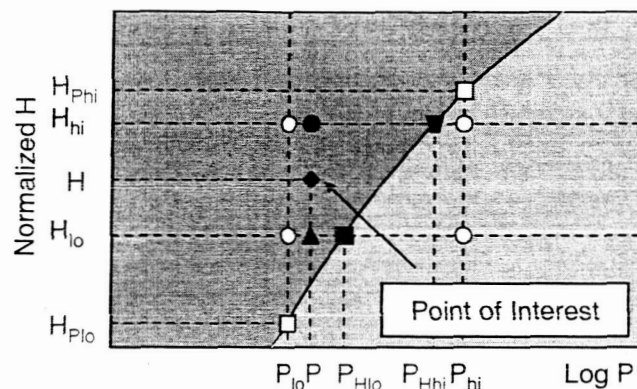


Figure 33. Saturated Vapor Case 4 – Sub-case 1

#### Sub-case 2

The second sub-case, where  $P$  is greater than or equal to  $P_{Hlo}$ , is shown in Figure 34. The properties for  $H_{hi}$  (shaded circle) are calculated by interpolating between the  $P_{Hhi}$  saturated properties (shaded trapezoid) and the top left data point (at  $P_{lo}$  and  $H_{hi}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the saturated properties for  $P$  (shaded triangle) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{hi}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy  $H$ .

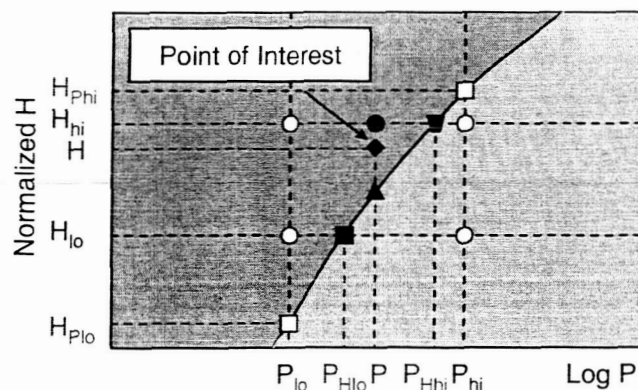


Figure 34. Saturated Vapor Case 4 – Sub-case 2

#### Saturated Vapor Case 5

Saturated Vapor Case 5 as defined in Figure 15 is shown in Figure 35 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and three of the four data points can not be used in the interpolation process because they lie inside the saturation envelope. To interpolate for the point of interest, the top right data

(shaded diamond) are determined by interpolating between the  $H_{hi}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy ( $H$ ).

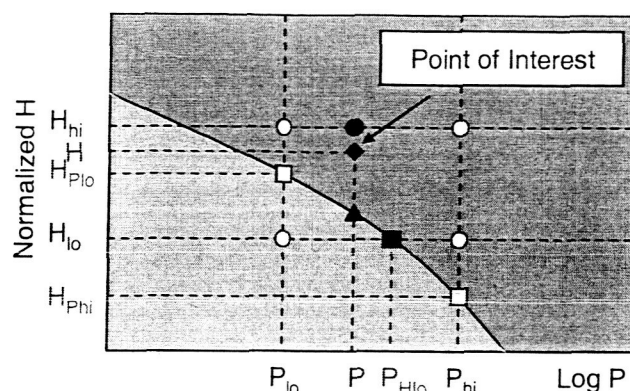


Figure 36. Saturated Vapor Case 6 – Sub-case 1

### Sub-case 2

The second sub-case, where  $P$  is greater than or equal to  $P_{Hlo}$ , is shown in Figure 37. The saturated properties for  $P_{Hlo}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{Hlo}$  as input. Now the properties for  $H_{lo}$  (shaded triangle) are calculated by interpolating between the  $P_{Hlo}$  saturated properties (shaded square) and the bottom right data point (at  $P_{hi}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the properties for  $H_{hi}$  (shaded circle) are obtained by interpolating between the top two data points ( $P_{lo}$  and  $P_{hi}$  at  $H_{hi}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded triangle and shaded circle) using the input enthalpy  $H$ .

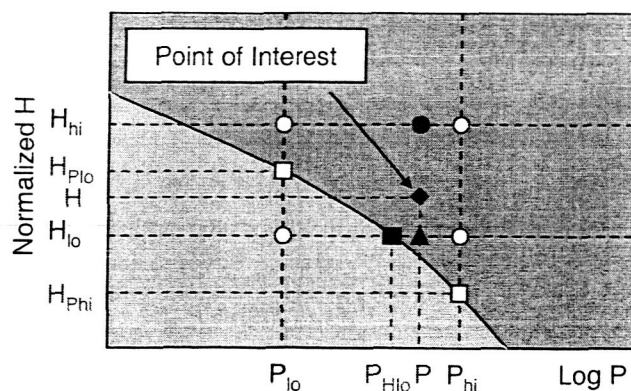


Figure 37. Saturated Vapor Case 6 – Sub-case 2

### Saturated Vapor Case 7

Saturated Vapor Case 7 as defined in Figure 15 is shown in Figure 38 within the state space. The standard interpolation rectangle is formed by four unshaded circles (from Property Table 2), and the two left data points ( $P_{lo}$  at  $H_{lo}$  and at  $H_{hi}$ ) can not be used in the interpolation process because they lie inside the saturation envelope. First, it is necessary to determine the pressure ( $P_{Hlo}$ ) at which the saturated vapor line crosses the lower enthalpy line ( $H_{lo}$ ) of the

interpolation rectangle. The first step in determining  $P_{H_{lo}}$  is to get saturated enthalpies for  $P_{lo}$  ( $H_{P_{lo}}$ ) and  $P_{hi}$  ( $H_{P_{hi}}$ ) using Property Table 1; then by using Equation 1,  $P_{H_{lo}}$  can be calculated by interpolating between  $H_{P_{lo}}$  and  $H_{P_{hi}}$  using  $H_{lo}$  as input. Similarly,  $P_{H_{hi}}$  is the pressure at which the saturated vapor line crosses the higher enthalpy line ( $H_{hi}$ ) of the interpolation rectangle, and it can be obtained by interpolating between  $H_{P_{lo}}$  and  $H_{P_{hi}}$  using  $H_{hi}$  as input. Next, the input pressure ( $P$ ) is compared to  $P_{H_{lo}}$ , and the comparison defines two sub-cases within Saturated Vapor Case 7. One sub-case is where  $P$  is less than  $P_{H_{lo}}$ , and the other sub-case is where  $P$  is greater than or equal to  $P_{H_{lo}}$ .

### Sub-case 1

The first sub-case, where  $P$  is less than  $P_{H_{lo}}$ , is shown in Figure 38. The saturated properties for  $P_{H_{hi}}$  (shaded trapezoid) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{H_{hi}}$  as input. Now the properties for  $H_{hi}$  (shaded circle) are calculated by interpolating between the  $P_{H_{hi}}$  saturated properties (shaded trapezoid) and the top right data point (at  $P_{hi}$  and  $H_{hi}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the saturated properties for  $P$  (shaded triangle) are obtained by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{hi}$  properties (shaded circle) and the saturated properties for  $P$  (shaded triangle) using the input enthalpy ( $H$ ).

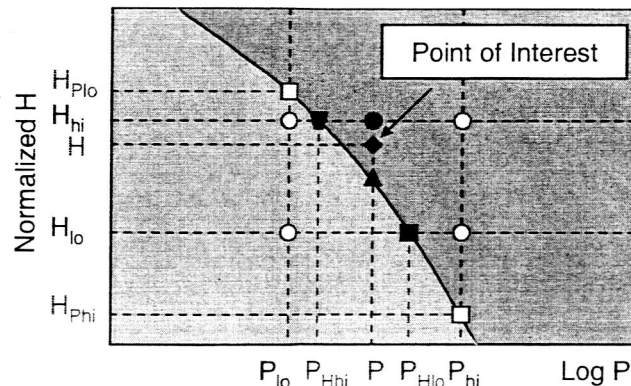


Figure 38. Saturated Vapor Case 7 – Sub-case 1

### Sub-case 2

The second sub-case, where  $P$  is greater than or equal to  $P_{H_{lo}}$ , is shown in Figure 39. The saturated properties for  $P_{H_{lo}}$  (shaded square) are computed by interpolating between the  $P_{lo}$  and  $P_{hi}$  saturated properties (unshaded squares) using  $P_{H_{lo}}$  as input. Now the properties for  $H_{lo}$  (shaded triangle) are calculated by interpolating between the  $P_{H_{lo}}$  saturated properties (shaded square) and the bottom right data point (at  $P_{hi}$  and  $H_{lo}$ ) from Property Table 2 using the input pressure  $P$ . Similarly, the properties for  $H_{hi}$  (shaded circle) are obtained by interpolating between the  $P_{H_{hi}}$  saturated properties (shaded trapezoid) and the top right data point (at  $P_{hi}$  and  $H_{hi}$ ) from Property Table 2 using  $P$  as input. Finally the properties of interest at  $P$  and  $H$  (shaded diamond) are determined by interpolating between the  $H_{lo}$  and  $H_{hi}$  properties (shaded triangle and shaded circle) using the input enthalpy  $H$ .

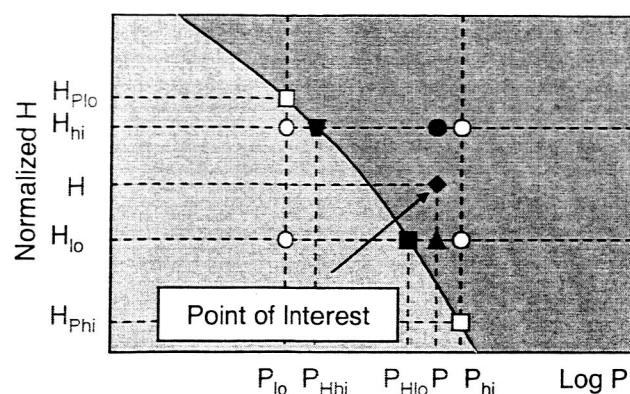


Figure 39. Saturated Vapor Case 7 – Sub-case 2

### PROPERTY TABLES VERSUS PROPERTY PACKAGES

As stated earlier, the main reason for using property tables instead of property packages within a computer simulation is to save computational time, but how much time is actually saved by using property tables? To answer this question, a simple model was built where there is only one control volume (node), and the thermodynamic properties for that volume were determined 50,000 times by using the same pressure and enthalpy each time. When property tables were used, it only took about 13 seconds to get the oxygen properties; however, it took almost 25 minutes (24 minutes and 54 seconds) when GASPAK was used to perform the same task. This significantly longer time when using the property package occurred because GASPAK was called within the computer simulation. There is no doubt that it would take a lot less time if GASPAK is used outside of the computer simulation, but the reason for building property tables is to use them within a computer simulation.

Now let us consider a mathematical model for a hypothetical facility used for component testing and see how long it would take for the model to run with property tables versus a property package. Let us assume that the model is comprised of 25 control volumes. Fifteen of those volumes have only one fluid, either a fuel or an oxidizer, and the other ten volumes have two fluids, a fuel and an inert fluid or an oxidizer and an inert fluid. First, let us consider the one-fluid volumes. In order to get property data for a one-fluid volume, the computer code used in this example calls a property package (or property tables) three times, so the total number of property calls for fifteen one-fluid volumes is 45 times. For a two-fluid volume, the computer code calls a property package (or property tables) six times, three times per fluid. Since there are two fluids, it is assumed that the fluids would mix together and become a homogeneous mixture, and in order to get reasonable properties for the mixture, it is assumed that on average it would take about three iterations. So a two-fluid volume would require a total of 18 property calls, and consequently, the total number of property calls for ten two-fluid volumes is 180 times. Hence, the total number of property calls for this twenty-five-volume example is 225 times. Now let us assume that it would take, on average, three iterations for the overall system model to converge for each time step, so the total number of property calls per converged time step is 675 times. Finally, if the simulation is run for five seconds with a time step of 0.001 second then the total number of property calls is 3,375,000 times. As noted earlier for every 50,000 property calls, it would take about 13 seconds when using property tables and about 25 minutes when using a property package. So for this example, it would take about 15 minutes versus 28 hours to finish a five-second simulation. This example is illustrated in Table 2, and it represents a fairly typical mathematical modeling situation.

Table 2. Twenty-Five-Volume Model

		One-Fluid Volume	Two-Fluid Volume
One Time Step	Number of property calls for one volume	3	6
	Homogeneous mixture - Number of iterations	1	3
	Total number of property calls for one volume	3	18
	Number of volumes in the model	15	10
	Number of property calls for each volume type	45	180
	Total number of property calls for the model	225	
Number of iterations for each time step to converge		3	
Total number of property calls for each time step		675	
Number of time steps (five-second simulation, 0.001 second time step)		5000	
Total number of property calls for the simulation		3,375,000	
		Property Tables	Property Package (GASPAK)
Time required for every 50,000 property calls		13 sec.	24 min. and 54 sec.
Total simulation time		~15 min.	~28 hrs.

### SUMMARY AND CONCLUSIONS

A method to define thermodynamic fluid property tables and a table interpolation technique is described to allow efficient and accurate predictions of properties in computer-based models of thermodynamic systems. The first part of the paper shows how efficient and usable property tables were generated, with the minimum number of data points and a maximum interpolated property error, using an aerospace standard property package. The number of data points and their grid spacing in the tables were indirectly determined from the specified acceptable error between interpolated values from the tables and values obtained from the property package. The second part of the paper describes how thermodynamic properties were retrieved from the tables. For the most part, the interpolation method used is straightforward linear interpolation between property table grid points. However, for fourteen cases near the saturation lines, a special interpolation method is presented which enables obtaining accurate property data to handle both sub-critical and supercritical regions.

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### REFERENCES

- [1] Adebiyi, G. A., and Russell, L. D., 1992, "Computer Codes for the Thermodynamic Properties of Common Substances Encountered in Engineering Applications," ASME Conference Proceedings, HTD-Vol. 225, pp. 1-7.

- [2] Adebiyi, G. A., 2003, "Formulations for the Thermodynamic Properties of Pure Substances," ASME Conference Proceedings.
- [3] Hendricks, R. C., Baron, A. K., and Peller, I. C., "GASP - A Computer Code for Calculating the Thermodynamic and Transport Properties for Ten Fluids: Parahydrogen, Helium, Neon, Methane, Nitrogen, Carbon Monoxide, Oxygen, Fluorine, Argon, and Carbon Dioxide," NASA Technical Note D-7808, Feb. 1975.
- [4] Lemmon, E. W., Peskin, A. P., McLinden, M. O., and Friend, D. G., "NIST Thermodynamic and Transport Properties of Pure Fluids – NIST Pure Fluids," Version 5.0, September 2000, Physical and Chemical Properties Division, National Institute of Standards and Technology.
- [5] Russell, L. D., and Adebiyi, G. A., 1993, *Classical Thermodynamics*, Saunders College Publishing: New York, New York.